The 13th International Workshop on the
Algorithmic Foundations of Robotics

Universidad Politécnica de Yucatán
Mérida, México, December 9, 2018

Path Planning for Ellipsoidal Robots via Generalized Closed-form Minkowski Operations

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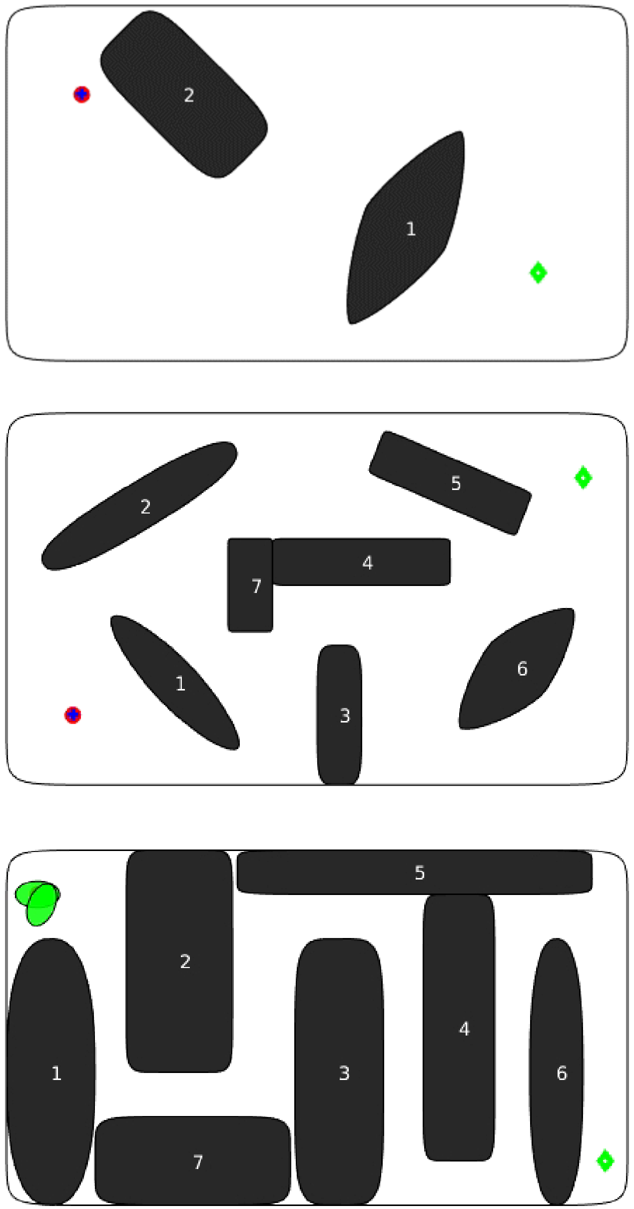
Introduction

- Probabilistic path planners are generally very efficient
  - PRM, RRT and variants
  - “Sample-Checking-Resample” procedure
  - Advantageous in high dimensional problems
  - Probabilistic complete
- Prohibitively expensive in a “Narrow Passage” problem
  - Hard to sample a collision-free path through a narrow corridor
- Can we reduce collision detection computation?
  - Combinatorial methods help
  - Works well for narrow passage problems
- Ellipsoidal robot
  - Simple and clean characterization
  - Closed-form Minkowski sum and difference



Mathematical Preliminary

- The closed-form Minkowski operations between an ellipsoid and any convex differentiable surface in $\mathbb{R}^n$

Original surface

Implicit: $\Phi(\boldsymbol{x}) = 1$
Parametric: $\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{\varphi})$

Shrinking to sphere

Affine transform into “shrunk space”

$$T = R_2 \Lambda(\boldsymbol{a}_2 / r) R_2^T$$
$$\boldsymbol{x}' = T\boldsymbol{x}$$

Offset touching surface

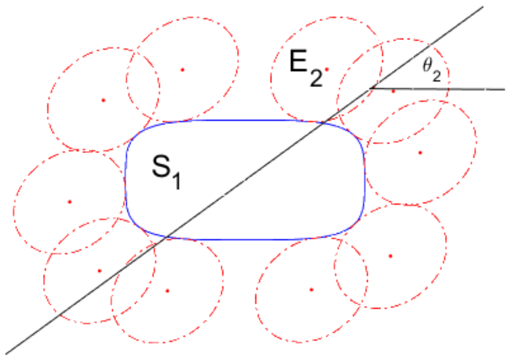
Offset radius r along outward normal

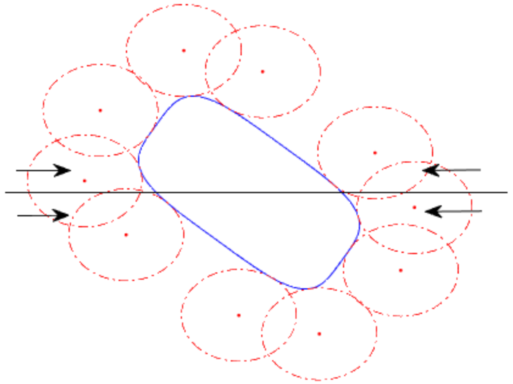
$$\boldsymbol{n} = \frac{\nabla\Phi(\boldsymbol{x}')}{\|\nabla\Phi(\boldsymbol{x}')\|}$$
$$\boldsymbol{x}'_{ofs} = \boldsymbol{x}' \pm r\boldsymbol{n}$$

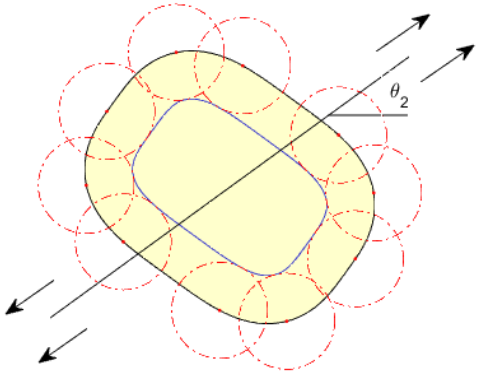
Stretching back

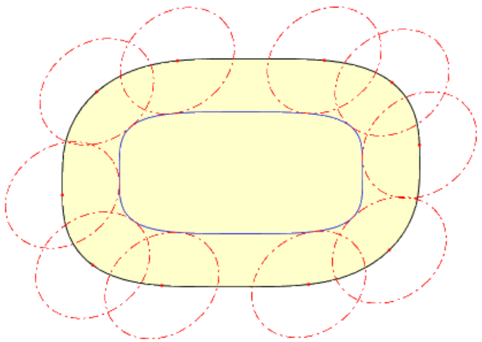
Preserves convexity

$$\boldsymbol{x}_{ofs} = T^{-1}\boldsymbol{x}'_{ofs}$$





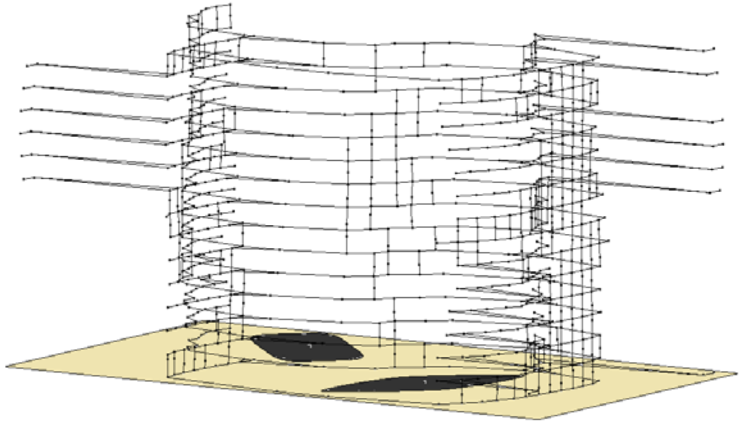




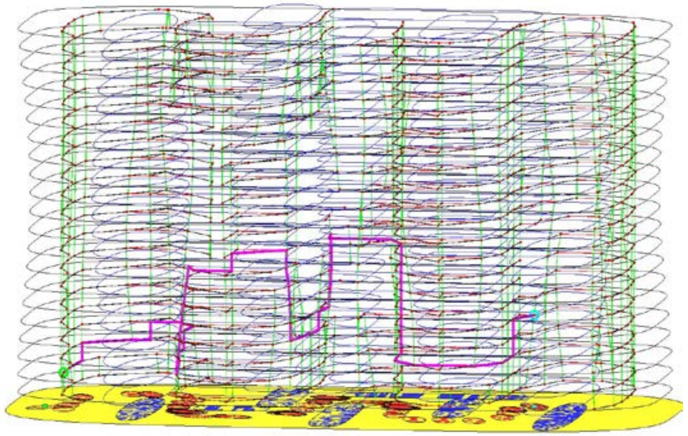
Highway RoadMap Planner Overview

- Constructions of C-layers: rotation discretization
- At each C-layer:
  - C-obstacle boundary computations: Closed-form Minkowski operations
  - Collision free subgraphs: Sweep line approach for cell decomposition
- Vertex connections between adjacent C-layers
  - Local C-space idea: the Kinematics of Containment for ellipsoids
- Graph search for a valid path

Sparse map

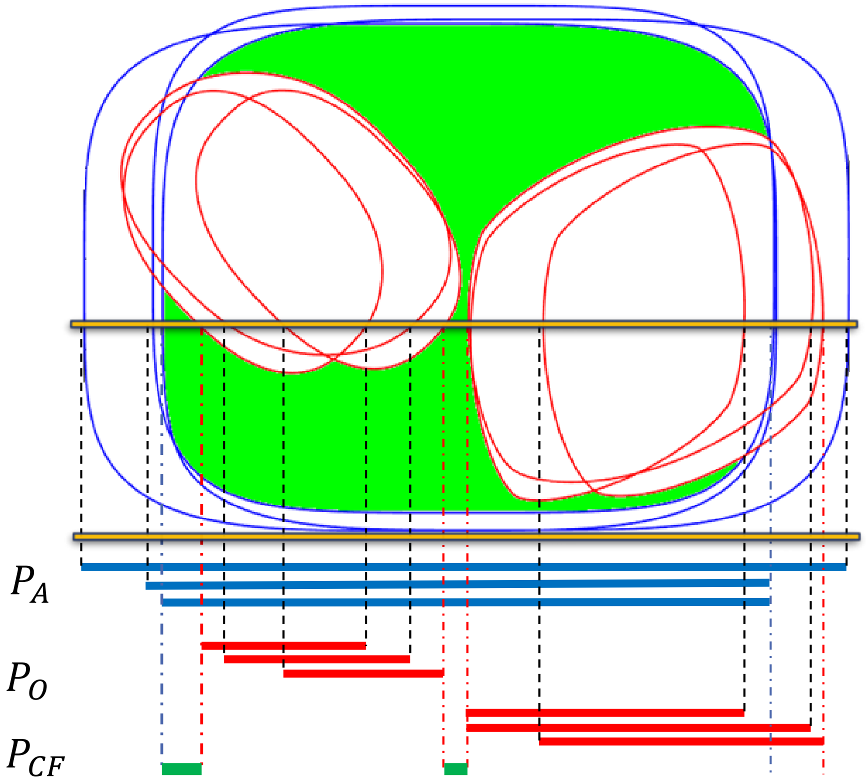


Maze map

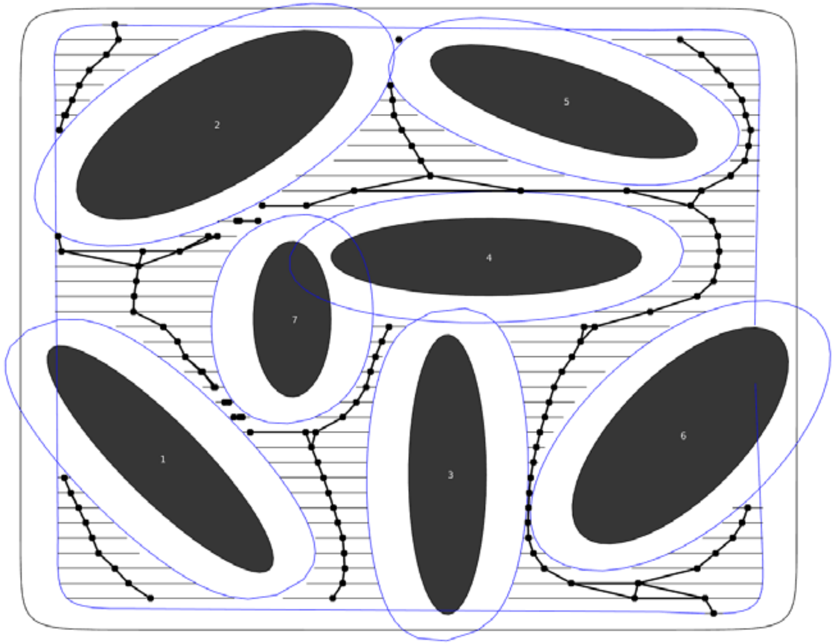


Cell Decomposition at each C-layer

- Sweep-Line process for detecting collision-free subspace
  - Each line: $P_{CF} = \bigcap_{i=1}^{n \times n_A} P_{Ai} - \bigcup_{j=1}^{n \times n_O} P_{Oj}$



- Subgraph at each C-layer
  - Middle point on P_{CF} as collision-free vertex
  - Vertex connections within one C-layer



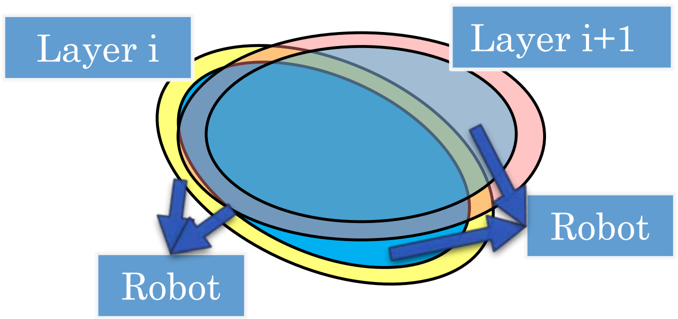
$\boldsymbol{P_A}$: Arena line segments;
$\boldsymbol{P_O}$: Obstacle line segments;
$\boldsymbol{P_{CF}}$: Collision-free segments;

$\boldsymbol{n_A}$: Number of arenas;
$\boldsymbol{n_O}$: Number of obstacles;
$\boldsymbol{n}$: Number of robot rigid parts

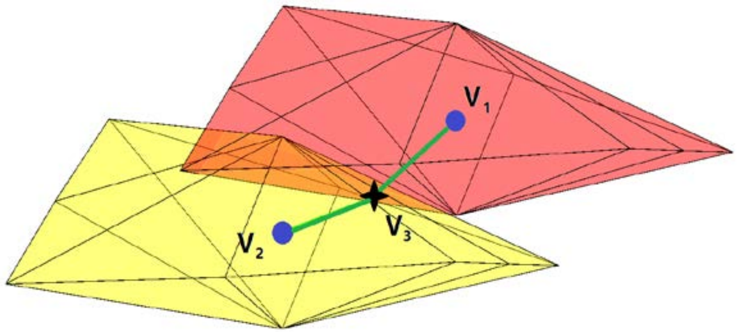
Vertex connections between adjacent C-layers

- Enclose the robot by a slightly larger ellipsoid
- Compute Minkowski Sum/Difference using the larger ellipsoid
  - Robot can move inside free of collision
  - Description of such motion formulates a "Local C-space" for the vertex
  - **Convex Lower bound for allowable motion of an ellipsoid contained in another**
- New vertex at the intersection between the local c-space of two vertices
  - Connect new vertex with the two original vertices respectively
- Avoid traditional collision checking computations

Illustration of the problem



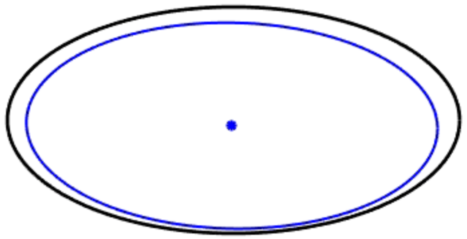
Local C-space for vertex connections between different layers



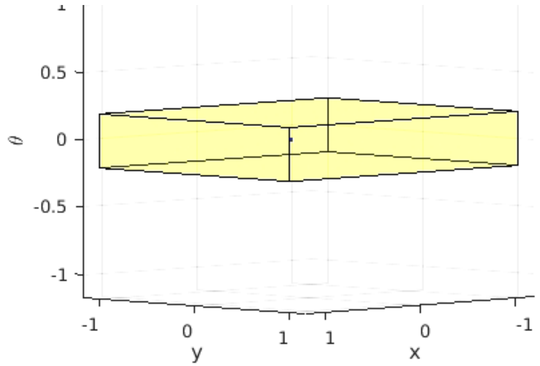
Local C-space: the Kinematics of Containment Idea

- Algebraic Condition of Containment for n-dimensional Ellipsoids:
  - Exact: $(R_a\Lambda(\boldsymbol{a})\boldsymbol{u} + \boldsymbol{t}_a)^T\Lambda^{-2}(\boldsymbol{b})(R_a\Lambda(\boldsymbol{a})\boldsymbol{u} + \boldsymbol{t}_a) \leq 1$
  - First-order Approximation (for frustrated motions): $R_a \approx \mathbb{I} + \widehat{\omega}_a,\ \boldsymbol{\xi} = [\boldsymbol{\omega}^T, \boldsymbol{t}^T]^T \in \mathbb{R}^{n(n+1)/2}$
  - Require: $C_{\boldsymbol{u}}(\boldsymbol{\xi}) = \boldsymbol{\xi}^T H(\boldsymbol{u})\boldsymbol{\xi} + \boldsymbol{h}^T(\boldsymbol{u})\boldsymbol{\xi} + c(\boldsymbol{u}) \leq 1 \Leftrightarrow \max\limits_{\forall \boldsymbol{u}_i} C_i(\boldsymbol{\xi}) \leq 1$
- Convexity of the First-order Algebraic Condition of Containment
  - Valid configurations stays in the convex hull of some extreme configurations
- Polyhedron C-space as the Convex Lower Bound

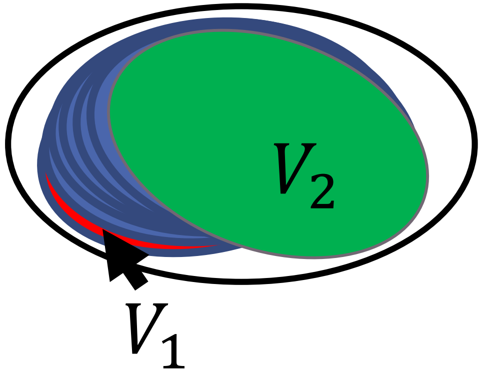
Euclidean space



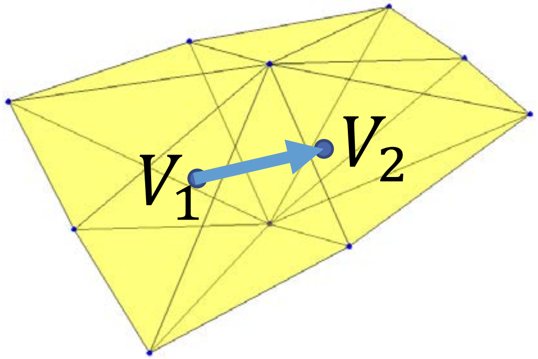
C-space



Euclidean space



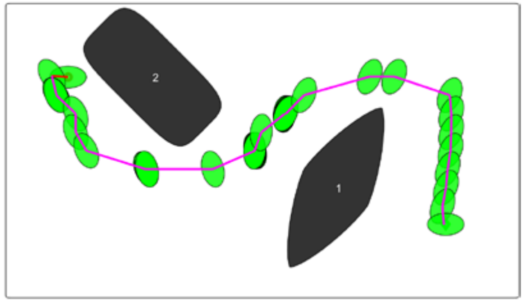
C-space

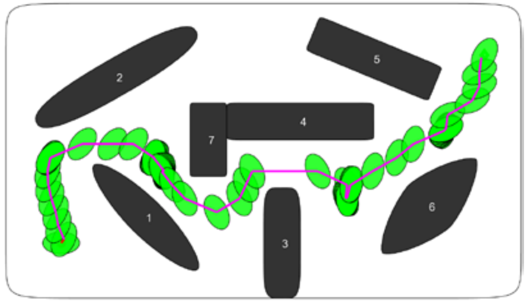


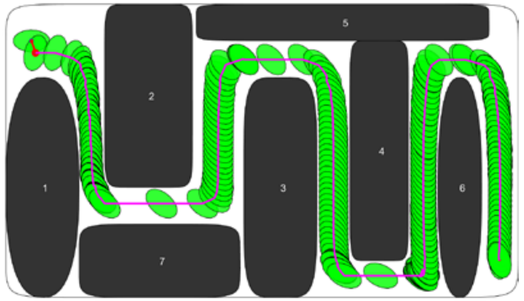
Experiments in $SE(2)$

- Parameters of Highway RoadMap planner (Superelliptical obstacles)

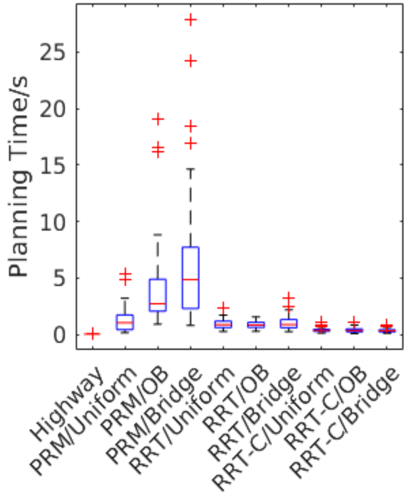
Map Type	# of C-layers	# of sweep lines	# of vertices	# of edges
Sparse	14	10	493	572
Cluttered	14	25	2009	2547
Maze	55	30	9782	13450



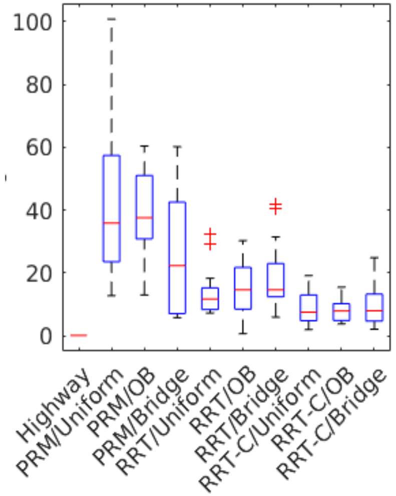




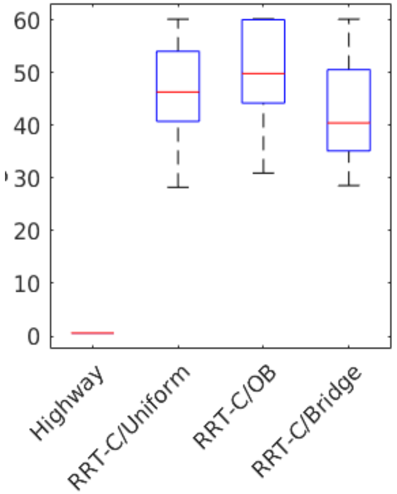
- Running time comparisons with sample-based planners from OMPL
  - Planners: PRM, RRT, RRT-Connect
  - Sampling methods: Uniform, Obstacle-based (OB), Bridge test
  - Collision detection: GJK method, discrete point set on the boundary



Sparse



Cluttered



Maze

Conclusion

- Proposed the closed-form Minkowski sum/difference between an ellipsoid and any convex differentiable surface embedded in $\mathbb{R}^n$
- Extended the Highway RoadMap planner
  - Superquadrics obstacles
  - Novel vertex connection method between adjacent C-layers
- Implemented in C++ and benchmarked with sample-based planners from OMPL
  - Compared computational time
  - Compared with different sampling methods
  - Highway RoadMap performs more efficiently, especially in "narrow passage" problem
- Have potential to build hybrid planners with sample-based methods
  - Deal with high-dimensional problems with narrow corridors

Dirty Laundry

- Geometry for robot and environment
  - The curvature constraint for Minkowski difference
    - In shrunk space: radius of curvature of S_1 at every point should be larger than the radius r
    - How to deal with it in practice: Set environment limit, fill up boundary with obstacles
  - How the shape of ellipsoid affects the performance? The impact of inflation?
    - Both will affect the volume of Local C-space, therefore the computations of middle vertex
    - Related to rotation resolution: C-layer distance smaller than largest rotational angle along each rotational axis

- Implementation and experiment details
  - Values of the discretization, i.e. number of C-layers and sweep lines
  - Comparisons for different sampling methods

- *Work in progress*: Implementations of the challenging $SE(3)$ case
  - Rotation discretization: uniform random, uniform grid on $SO(3)$
  - Local C-space: 6D convex polyhedron

Thanks!

- Special thanks to
  - Funding Agencies: US National Science Foundation & US Office of Naval Research
  - Dr. Fan Yang, Mr. Thomas W. Mitchel and Mr. Zeyi Wang for useful discussions
  - Members in Robot and Protein Kinematics Lab